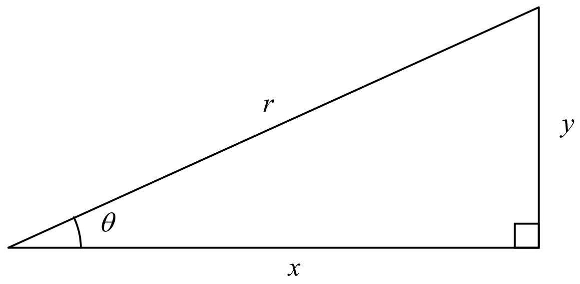
図形と計量 2　三角比の相互関係

三角比



直角三角形の斜辺の長さをr，底辺の長さをx，高さをy，斜辺と底辺のなす角をθとすると，

$$\cos\theta = \frac{x}{r},\quad \sin\theta = \frac{y}{r},\quad \tan\theta = \frac{y}{x}$$

三角比の相互関係

1. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$　**2.** $\sin^2\theta + \cos^2\theta = 1$　**3.** $1+\tan^2\theta = \dfrac{1}{\cos^2\theta}$　**4.** $1+\dfrac{1}{\tan^2\theta} = \dfrac{1}{\sin^2\theta}$

導き方の例

1.　$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\dfrac{y}{x}\cdot\dfrac{x}{r} = \dfrac{y}{r}$　すなわち $\tan\theta\cos\theta = \sin\theta$　$\therefore \tan\theta = \dfrac{\sin\theta}{\cos\theta}$

2.　$\sin^2\theta + \cos^2\theta = 1$

三平方の定理より，$x^2 + y^2 = r^2$

両辺をr^2で割ると，$\dfrac{x^2}{r^2} + \dfrac{y^2}{r^2} = 1$

よって，$\left(\dfrac{x}{r}\right)^2 + \left(\dfrac{y}{r}\right)^2 = 1$　すなわち $\sin^2\theta + \cos^2\theta = 1$

3.　$1+\tan^2\theta = \dfrac{1}{\cos^2\theta}$

三平方の定理より，$x^2 + y^2 = r^2$

両辺をx^2で割ると，$1 + \dfrac{y^2}{x^2} = \dfrac{r^2}{x^2}$

よって，$1 + \left(\dfrac{y}{x}\right)^2 = \left(\dfrac{r}{x}\right)^2$　すなわち $1+\tan^2\theta = \dfrac{1}{\cos^2\theta}$　$\left(\because \dfrac{r}{x} = \dfrac{1}{\frac{x}{r}} = \dfrac{1}{\cos\theta}\right)$

4. $1+\dfrac{1}{\tan^2\theta}=\dfrac{1}{\sin^2\theta}$

三平方の定理より， $x^2+y^2=r^2$

両辺を y^2 で割ると， $\dfrac{x^2}{y^2}+1=\dfrac{r^2}{y^2}$

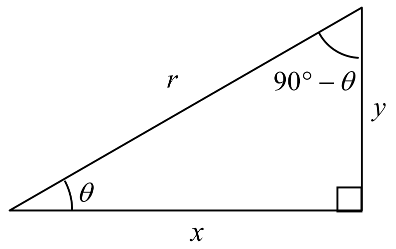
よって， $1+\left(\dfrac{x}{y}\right)^2=\left(\dfrac{r}{y}\right)^2$ すなわち $1+\dfrac{1}{\tan^2\theta}=\dfrac{1}{\sin^2\theta}$

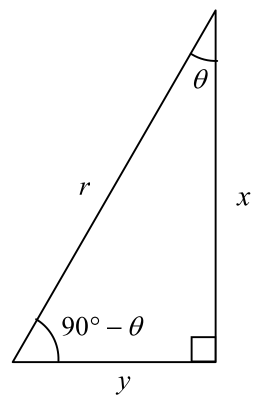
$\left(\because \dfrac{x}{y}=\dfrac{1}{\frac{y}{x}}=\dfrac{1}{\tan\theta},\ \dfrac{r}{x}=\dfrac{1}{\frac{x}{r}}=\dfrac{1}{\cos\theta}\right)$

$90°-\theta$ の三角比

$\sin(90°-\theta)=\cos\theta \quad \cos(90°-\theta)=\sin\theta \quad \tan(90°-\theta)=\dfrac{1}{\tan\theta}$

導き方の例





$\sin\theta=\dfrac{y}{r}$

$\cos\theta=\dfrac{x}{r}$

$\tan\theta=\dfrac{y}{x}$

$\sin(90°-\theta)=\dfrac{x}{r}$

$\cos(90°-\theta)=\dfrac{y}{r}$

$\tan(90°-\theta)=\dfrac{x}{y}$

よって， $\sin(90°-\theta)=\cos\theta$， $\cos(90°-\theta)=\sin\theta$， $\tan(90°-\theta)=\dfrac{1}{\tan\theta}$

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(1)

$$\sin^2 40°`+\sin^2 50° = \sin^2 40° + \sin^2(90° - 40°)$$
$$= \sin^2 40° + \{\sin(90° - 40°)\}^2$$
$$= \sin^2 40° + (\cos 40°)^2$$
$$= \sin^2 40° + \cos^2 40°$$
$$= 1$$

(2)

$$\tan 35° \tan 55° - \tan 15° \tan 75° = \tan 35° \tan(90° - 35°) - \tan 15° \tan(90° - 15°)$$
$$= \tan 35° \cdot \frac{1}{\tan 35°} - \tan 15° \cdot \frac{1}{\tan 15°}$$
$$= 1 - 1$$
$$= 0$$

(3)

$$(\sin 70° + \sin 20°)^2 - 2\tan 70° \cos^2 70° = \{\sin 70° + \sin(90° - 70°)\}^2 - 2 \cdot \frac{\sin 70°}{\cos 70°} \cdot \cos^2 70°$$
$$= (\sin 70° + \cos 70°)^2 - 2\sin 70° \cos 70°$$
$$= \sin^2 70° + 2\sin 70° \cos 70° + \cos^2 70° - 2\sin 70° \cos 70°$$
$$= \sin^2 70° + \cos^2 70°$$
$$= 1$$

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(1)

$$(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = (\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta) + (\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta)$$
$$= 2(\sin^2\theta + \cos^2\theta)$$
$$= 2$$

(2)

$$(1 - \sin\theta)(1 + \sin\theta) - \frac{1}{1 + \tan^2\theta} = 1 - \sin^2\theta - \frac{1}{\frac{1}{\cos^2\theta}}$$
$$= \cos^2\theta - \cos^2\theta$$
$$= 0$$

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$$\begin{aligned}\sin^4\theta-\cos^4\theta&=(\sin^2\theta+\cos^2\theta)(\sin^2\theta-\cos^2\theta)\\&=1\cdot\{\sin^2\theta-(1-\sin^2\theta)\}\\&=2\sin^2\theta-1\end{aligned}$$

$$\begin{aligned}\sin^4\theta-\cos^4\theta&=(\sin^2\theta+\cos^2\theta)(\sin^2\theta-\cos^2\theta)\\&=1\cdot\{(1-\cos^2\theta)-\cos^2\theta\}\\&=1-2\cos^2\theta\end{aligned}$$