三角関数 7　加法定理の応用

297

$$
\begin{aligned}
\text{左辺} &= \cos 3\alpha + \sin 3\alpha \\
&= -3\cos\alpha + 4\cos^3\alpha + 3\sin\alpha - 4\sin^3\alpha \\
&= -3(\cos\alpha - \sin\alpha) + 4(\cos^3\alpha - \sin^3\alpha) \\
&= -3(\cos\alpha - \sin\alpha) + 4(\cos\alpha - \sin\alpha)(\cos^2\alpha + \cos\alpha\sin\alpha + \sin^2\alpha) \\
&= -3(\cos\alpha - \sin\alpha) + 4(\cos\alpha - \sin\alpha)(1 + \cos\alpha\sin\alpha) \\
&= (\cos\alpha - \sin\alpha)(-3 + 4 + 4\cos\alpha\sin\alpha) \\
&= (\cos\alpha - \sin\alpha)(1 + 2\sin 2\alpha) \\
&= \text{右辺}
\end{aligned}
$$

298　三角関数の有理関数化（数学Ⅲ積分と関連）

$1+\tan^2\alpha = \dfrac{1}{\cos^2\alpha}$ より，　$\cos^2\alpha = \dfrac{1}{1+\tan^2\alpha}$　　$\therefore \cos^2\alpha = \dfrac{1}{1+t^2}$

$$
\begin{aligned}
\sin 2\alpha &= 2\sin\alpha\cos\alpha \\
&= \frac{2\sin\alpha\cos\alpha}{1} \\
&= \frac{2\sin\alpha\cos\alpha}{\cos^2\alpha + \sin^2\alpha} \\
&= \frac{\cos^2\alpha \cdot 2 \cdot \dfrac{\sin\alpha}{\cos\alpha}}{\cos^2\alpha\left(1 + \dfrac{\sin^2\alpha}{\cos^2\alpha}\right)} \\
&= \frac{2 \cdot \dfrac{\sin\alpha}{\cos\alpha}}{1 + \left(\dfrac{\sin\alpha}{\cos\alpha}\right)^2} \\
&= \frac{2\tan\alpha}{1+\tan^2\alpha}
\end{aligned}
$$

$$\therefore \sin 2\alpha = \frac{2t}{1+t^2}$$

$$
\begin{aligned}
\cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\
&= \frac{\cos^2\alpha - \sin^2\alpha}{\cos^2\alpha + \sin^2\alpha} \\
&= \frac{\cos^2\alpha\left(1 - \dfrac{\sin^2\alpha}{\cos^2\alpha}\right)}{\cos^2\alpha\left(1 + \dfrac{\sin^2\alpha}{\cos^2\alpha}\right)}
\end{aligned}
$$

$$=\frac{1-\left(\frac{\sin\alpha}{\cos\alpha}\right)^2}{1+\left(\frac{\sin\alpha}{\cos\alpha}\right)^2}$$

$$=\frac{1-\tan^2\alpha}{1+\tan^2\alpha}$$

$$\therefore \cos 2\alpha=\frac{1-t^2}{1+t^2}$$

299

(1)

与式より，$\cos 2x-\cos x=0$

これと $\cos 2x=2\cos^2 x-1$ より，$2\cos^2 x-\cos x-1=0$ $\quad\therefore (2\cos x+1)(\cos x-1)=0$

ゆえに，$\cos x=-\frac{1}{2},\ 1$

$0\leq x<2\pi$ より，$x=0,\ \frac{2}{3}\pi,\ \frac{4}{3}\pi$

(2)

与式より，$\sin 2x-\cos x=0$

これと $\sin 2x=2\sin x\cos x$ より，$2\sin x\cos x-\cos x=0$ $\quad\therefore \cos x(2\sin x-1)=0$

ゆえに，$\cos x=0$ または $\sin x=\frac{1}{2}$

$0\leq x<2\pi$ より，$x=\frac{\pi}{6},\ \frac{\pi}{2},\ \frac{5}{6}\pi,\ \frac{3}{2}\pi$

(3)

$\cos 2x=2\cos^2 x-1$ より，$4\cos^2 x+4\cos x-3=0$ $\quad\therefore (2\cos x+3)(2\cos x-1)=0$

$2\cos x+3\neq 0$ より，$\cos x=\frac{1}{2}$

$0\leq x<2\pi$ より，$x=\frac{\pi}{3},\ \frac{5}{3}\pi$

(4)

$\cos 2x=2\cos^2 x-1$，$\sin 2x=2\sin x\cos x$ より，

$2\sin x\cos^2 x+2\sin x\cos x(1+\cos x)=0$ $\quad\therefore 2\sin x\cos x(2\cos x+1)=0$

ゆえに，$\sin x=0$ または $\cos x=0$ または $\cos x=-\frac{1}{2}$

$0\leq x<2\pi$ より，$x=0,\ \frac{\pi}{2},\ \frac{2}{3}\pi,\ \pi,\ \frac{4}{3}\pi,\ \frac{3}{2}\pi$

300

(1)

$\cos 2x = 1 - 2\sin^2 x$， $\cos 2x < \sin x$ より， $1 - 2\sin^2 x < \sin x$ $\therefore (2\sin x - 1)(\sin x + 1) > 0$

ゆえに， $\sin x > \dfrac{1}{2}$

これと $0 \leq x < 2\pi$ より， $\dfrac{\pi}{6} < x < \dfrac{5}{6}\pi$

(2)

$\cos 2x = -1 + 2\cos^2 x$， $\cos 2x \geq \cos^2 x$ より， $-1 + 2\cos^2 x \geq \cos^2 x$ $\therefore \cos^2 x \geq 1$

$0 \leq \cos^2 x \leq 1$ だから， $\cos x = -1,\ 1$

これと $0 \leq x < 2\pi$ より， $x = 0,\ \pi$

(3)

$\sin 2x = 2\sin x \cos x$， $\cos x + \sin 2x > 0$ より， $\cos x + 2\sin x \cos x > 0$ $\therefore \cos x(2\sin x + 1) > 0$

ゆえに $\cos x > 0,\ \sin x > -\dfrac{1}{2}$ または $\cos x < 0,\ \sin x < -\dfrac{1}{2}$

ここから，解法を 2 つに分ける。

解法 1：まともに求める

$\cos x > 0,\ \sin x > -\dfrac{1}{2}$ の解

$\left(0 \leq x < \dfrac{\pi}{2} \cup \dfrac{3}{2}\pi < x < 2\pi\right) \cap \left(0 \leq x < \dfrac{7}{6}\pi \cup \dfrac{11}{6}\pi < x < 2\pi\right)$ より， $0 \leq x < \dfrac{\pi}{2},\ \dfrac{11}{6}\pi < x < 2\pi$

$\cos x < 0,\ \sin x < -\dfrac{1}{2}$ の解

$\dfrac{\pi}{2} < x < \dfrac{3}{2}\pi \cap \dfrac{7}{6}\pi < x < \dfrac{11}{6}\pi$ より， $\dfrac{7}{6}\pi < x < \dfrac{3}{2}\pi$

以上より，

$0 \leq x < \dfrac{\pi}{2},\ \dfrac{7}{6}\pi < x < \dfrac{3}{2}\pi,\ \dfrac{11}{6}\pi < x < 2\pi$

解法 2：単位円を利用する

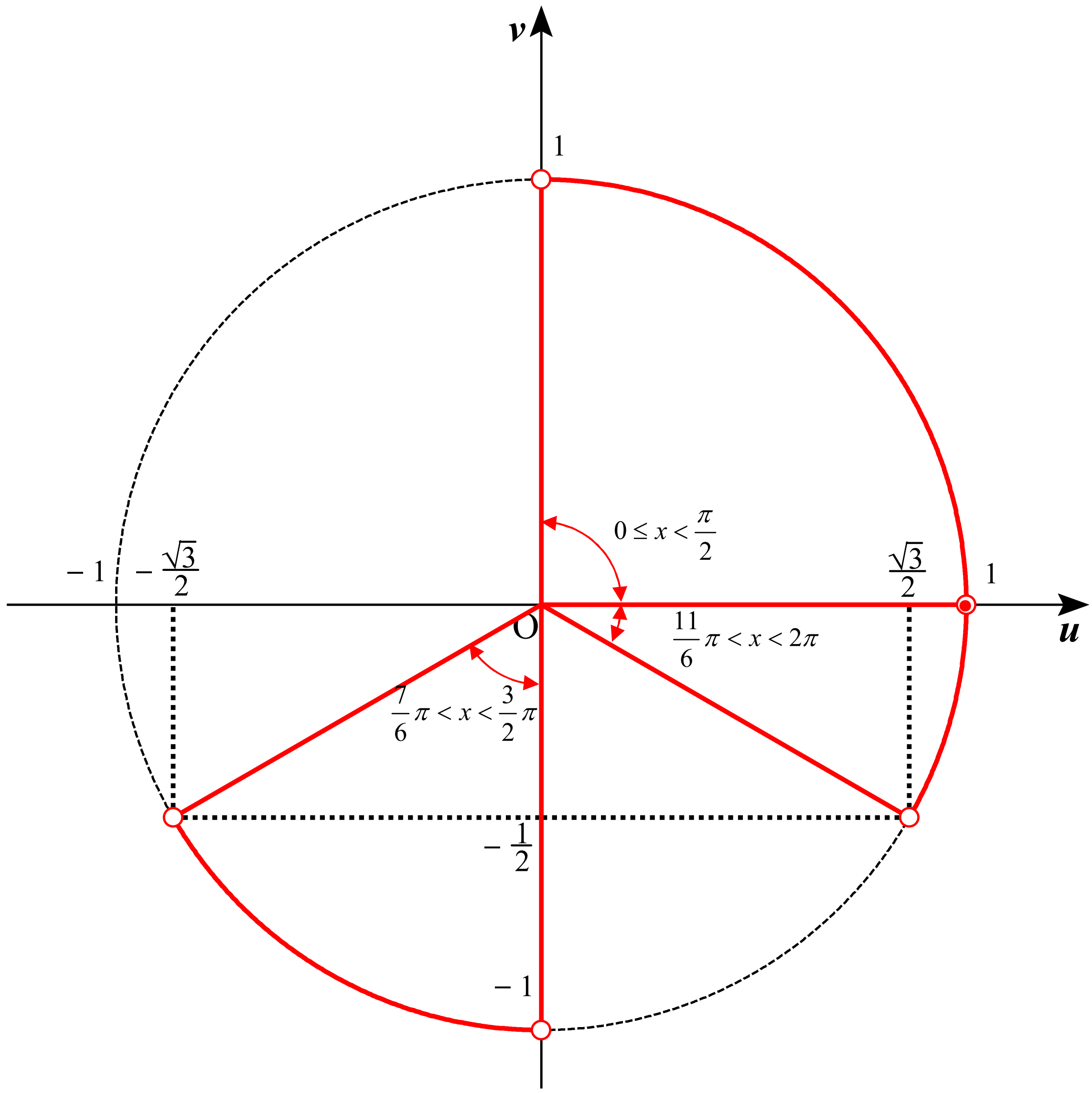
$\cos x = u,\ \sin x = v$ とおくと，

$u^2 + v^2 = 1,\ u > 0,\ v > -\dfrac{1}{2}$ または $u^2 + v^2 = 1,\ u < 0,\ v < -\dfrac{1}{2}$

これを図示すると次ページの図のようになる。

よって，

$0 \leq x < \dfrac{\pi}{2},\ \dfrac{7}{6}\pi < x < \dfrac{3}{2}\pi,\ \dfrac{11}{6}\pi < x < 2\pi$



v
u
1
−1
1
−1
O
$-\frac{\sqrt{3}}{2}$
$\frac{\sqrt{3}}{2}$
$-\frac{1}{2}$
$0 \leq x < \frac{\pi}{2}$
$\frac{11}{6}\pi < x < 2\pi$
$\frac{7}{6}\pi < x < \frac{3}{2}\pi$

301

$\cos 2x = 1 - 2\sin^2 x$ より，　$y = 2\sin^2 x + 2\sin x - 1 = 2\left(\sin x + \frac{1}{2}\right)^2 - \frac{3}{2}$

また，$-\frac{x}{2} \leq x \leq \frac{\pi}{2}$ より $-1 \leq \sin x \leq 1$

よって，

$\sin x = -\frac{1}{2}$ のとき，すなわち $x = -\frac{\pi}{6}$ のとき最小値 $-\frac{3}{2}$

$\sin x = 1$ のとき，すなわち $x = \frac{\pi}{2}$ のとき最大値 3

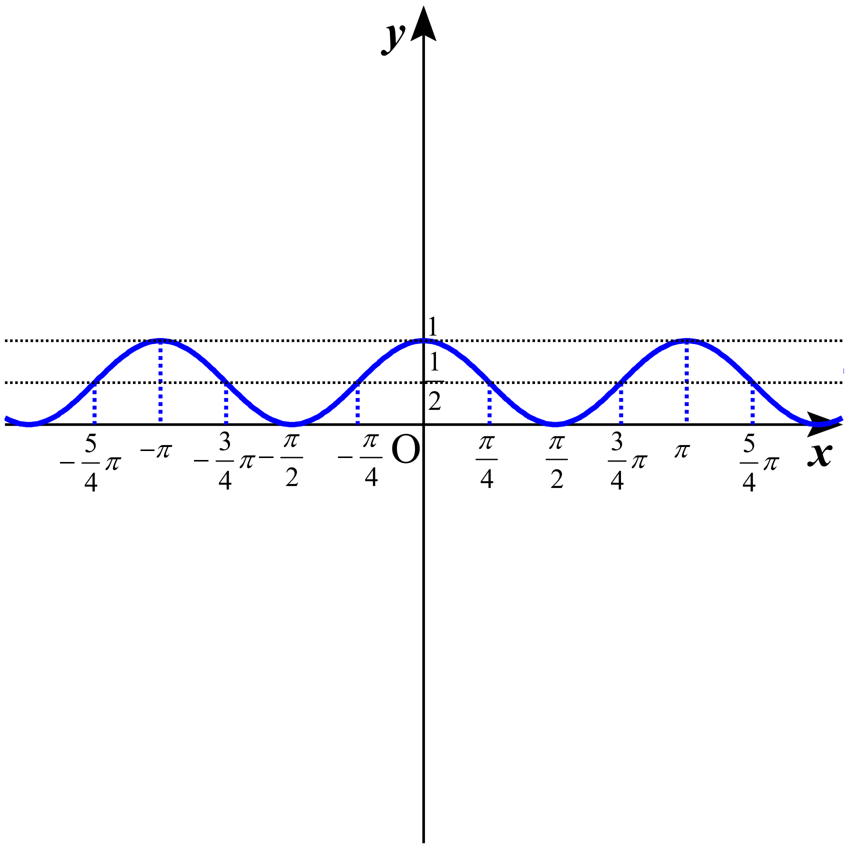
302

(1)

$\cos 2x = -1 + 2\cos^2 x$ より，　$y = \frac{1}{2}(1 + \cos 2x) = \frac{1}{2} + \frac{1}{2}\cos 2x$

グラフの描き方：$y = \frac{1}{2}\cos 2x$ のグラフを y 軸方向に $\frac{1}{2}$ 平行移動すればよい。

周期は $\cos 2x$ の周期と等しいから $\frac{2\pi}{2} = \pi$



(2)

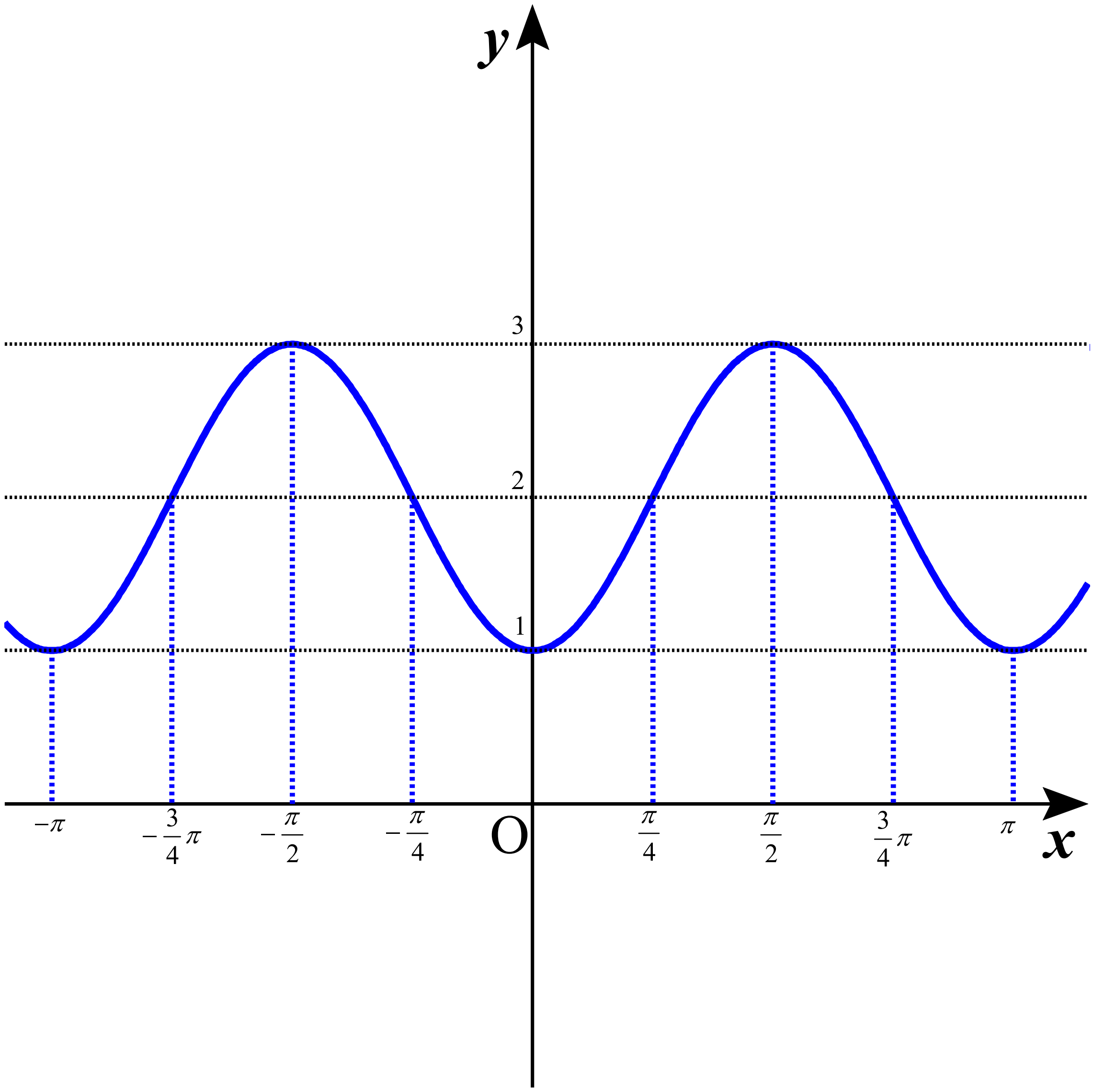
$$3\sin^2 x+\cos^2 x=3\cdot\frac{1-\cos 2x}{2}+\frac{1+\cos 2x}{2}$$
$$=-\cos 2x+2$$

より，　$y=-\cos 2x+2$

グラフの描き方

$y=\cos 2x$ のグラフを x 軸に関して対称移動すると $y=-\cos 2x$ のグラフになるから，これを y 軸方向に +2 平行移動すればよい。

周期は $\cos 2x$ の周期と等しいから $\frac{2\pi}{2}=\pi$



303

$\tan B\tan C=1$， $\tan B\tan C=\dfrac{\sin B}{\cos B}\cdot\dfrac{\sin C}{\cos C}$ より， $1=\dfrac{\sin B\sin C}{\cos B\cos C}$

$\therefore \cos B\cos C-\sin B\sin C=0$

これと

$$\begin{aligned}\cos B\cos C-\sin B\sin C&=\cos(B+C)\\&=\cos(\pi-A)\\&=-\cos A\end{aligned}$$

より， $\cos A=0$

$0<A<\pi$ だから， $A=\dfrac{\pi}{2}$

ゆえに，△ABC は∠A が直角である直角三角形である。

304

(1)

$$\begin{aligned}\sin 2\theta\cos 3\theta&=\frac{1}{2}\{\sin(2\theta+3\theta)+\sin(2\theta-3\theta)\}\\&=\frac{1}{2}\{\sin 5\theta+\sin(-\theta)\}\\&=\frac{1}{2}(\sin 5\theta-\sin\theta)\end{aligned}$$

(2)

$$\begin{aligned}\cos 4\theta\cos\theta&=\frac{1}{2}\{\cos(4\theta+\theta)+\cos(4\theta-\theta)\}\\&=\frac{1}{2}(\cos 5\theta+\cos 3\theta)\end{aligned}$$

(3)

$$\begin{aligned}\sin 3\theta\sin 5\theta&=\frac{1}{2}\{\cos(3\theta-5\theta)-\cos(3\theta+5\theta)\}\\&=\frac{1}{2}\{\cos(-2\theta)-\cos 8\theta\}\\&=\frac{1}{2}(\cos 2\theta-\cos 8\theta)\end{aligned}$$

305

$A=\alpha+\beta,\ B=\alpha-\beta$ とおくと，$\alpha=\dfrac{A+B}{2},\ \beta=\dfrac{A-B}{2}$ より，

$$\sin A+\sin B=2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A-\sin B=2\sin\frac{A-B}{2}\cos\frac{A+B}{2}$$

$$\cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A-\cos B=-2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

(1)

$$\begin{aligned}\sin 2\theta+\sin 4\theta&=2\sin\frac{2\theta+4\theta}{2}\cos\frac{2\theta-4\theta}{2}\\&=2\sin 3\theta\cos(-\theta)\\&=2\sin 3\theta\cos\theta\end{aligned}$$

(2)

$$\begin{aligned}\cos 4\theta-\cos 2\theta&=-2\sin\frac{4\theta+2\theta}{2}\sin\frac{4\theta-2\theta}{2}\\&=-2\sin 3\theta\sin\theta\end{aligned}$$

(3)

$$\begin{aligned}\cos 3\theta+\cos 2\theta&=2\cos\frac{3\theta+2\theta}{2}\cos\frac{3\theta-2\theta}{2}\\&=2\cos\frac{5\theta}{2}\cos\frac{\theta}{2}\end{aligned}$$

306

(1)

$$\begin{aligned}\sin 75^\circ \cos 15^\circ &= \frac{1}{2}\{\sin(75^\circ + 15^\circ) + \sin(75^\circ - 15^\circ)\} \\ &= \frac{1}{2}(\sin 90^\circ + \sin 60^\circ) \\ &= \frac{1}{2}\left(1 + \frac{\sqrt{3}}{2}\right) \\ &= \frac{2+\sqrt{3}}{4}\end{aligned}$$

(2)

$$\begin{aligned}\cos 75^\circ \sin 15^\circ &= \frac{1}{2}\{\sin(75^\circ + 15^\circ) - \sin(75^\circ - 15^\circ)\} \\ &= \frac{1}{2}(\sin 90^\circ - \sin 60^\circ) \\ &= \frac{1}{2}\left(1 - \frac{\sqrt{3}}{2}\right) \\ &= \frac{2-\sqrt{3}}{4}\end{aligned}$$

(3)

$$\begin{aligned}\sin 37.5^\circ \sin 7.5^\circ &= \frac{1}{2}\{\cos(37.5^\circ - 7.5^\circ) - \cos(37.5^\circ + 7.5^\circ)\} \\ &= \frac{1}{2}(\cos 30^\circ - \cos 45^\circ) \\ &= \frac{1}{2}\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\right) \\ &= \frac{\sqrt{3}-\sqrt{2}}{4}\end{aligned}$$

(4)

$$\begin{aligned}\sin 75^\circ - \sin 15^\circ &= 2\sin\frac{75^\circ - 15^\circ}{2}\cos\frac{75^\circ + 15^\circ}{2} \\ &= 2\sin 30^\circ \cos 45^\circ \\ &= 2\cdot\frac{1}{2}\cdot\frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2}}{2}\end{aligned}$$

(5)

$$\begin{aligned}\cos 15^\circ + \cos 105^\circ &= 2\cos\frac{15^\circ + 105^\circ}{2}\cos\frac{15^\circ - 105^\circ}{2}\\ &= 2\cos 60^\circ \cos(-45^\circ)\\ &= 2\cdot\frac{1}{2}\cdot\frac{\sqrt{2}}{2}\\ &= \frac{\sqrt{2}}{2}\end{aligned}$$

(6)

$$\begin{aligned}\cos 105^\circ - \cos 15^\circ &= -2\sin\frac{105^\circ + 15^\circ}{2}\sin\frac{105^\circ - 15^\circ}{2}\\ &= -2\sin 60^\circ \sin 45^\circ\\ &= -2\cdot\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}\\ &= -\frac{\sqrt{6}}{2}\end{aligned}$$

307

(1)

$$\begin{aligned}\cos 20^\circ \cos 40^\circ \cos 80^\circ &= \frac{1}{2}\{\cos(20^\circ + 40^\circ) + \cos(20^\circ - 40^\circ)\}\cos 80^\circ\\ &= \frac{1}{2}\{\cos 60^\circ + \cos(-20^\circ)\}\cos 80^\circ\\ &= \frac{1}{2}\left(\frac{1}{2} + \cos 20^\circ\right)\cos 80^\circ\\ &= \frac{1}{4}(\cos 80^\circ + 2\cos 20^\circ \cos 80^\circ)\\ &= \frac{1}{4}\{\cos 80^\circ + \cos(20^\circ + 80^\circ) + \cos(20^\circ - 80^\circ)\}\\ &= \frac{1}{4}\{\cos 80^\circ + \cos 100^\circ + \cos(-60^\circ)\}\\ &= \frac{1}{4}\left\{\cos 80^\circ + \cos(180^\circ - 80^\circ) + \frac{1}{2}\right\}\\ &= \frac{1}{4}\left(\cos 80^\circ - \cos 80^\circ + \frac{1}{2}\right)\\ &= \frac{1}{8}\end{aligned}$$

(2)

$$\begin{aligned}\sin 20^\circ + \sin 140^\circ + \sin 260^\circ &= 2\sin\frac{20^\circ + 140^\circ}{2}\cos\frac{20^\circ - 140^\circ}{2} + \sin 260^\circ \\ &= 2\sin 80^\circ \cos(-60^\circ) + \sin 260^\circ \\ &= \sin 80^\circ + \sin(360^\circ - 100^\circ) \\ &= \sin 80^\circ - \sin 100^\circ \\ &= \sin(180^\circ - 100^\circ) - \sin 100^\circ \\ &= \sin 100^\circ - \sin 100^\circ \\ &= 0\end{aligned}$$

または

$$\begin{aligned}\sin 20^\circ + \sin 140^\circ + \sin 260^\circ &= 2\sin\frac{20^\circ + 140^\circ}{2}\cos\frac{20^\circ - 140^\circ}{2} + \sin 260^\circ \\ &= 2\sin 80^\circ \cos(-60^\circ) + \sin 260^\circ \\ &= \sin 80^\circ + \sin(360^\circ - 100^\circ) \\ &= \sin 80^\circ - \sin 100^\circ \\ &= 2\sin\frac{80^\circ - 100^\circ}{2}\cos\frac{80^\circ + 100^\circ}{2} \\ &= -2\sin 40^\circ \cos 90^\circ \\ &= 0\end{aligned}$$

308

(1)

$$\begin{aligned}\cos(\alpha+\beta)\sin(\alpha-\beta) &= \frac{1}{2}[\sin\{(\alpha-\beta)+(\alpha+\beta)\} + \sin\{(\alpha-\beta)-(\alpha+\beta)\}] \\ &= \frac{1}{2}\{\sin 2\alpha + \sin(-2\beta)\} \\ &= \frac{1}{2}(\sin 2\alpha - \sin 2\beta)\end{aligned}$$

同様にして,

$$\cos(\beta+\gamma)\sin(\beta-\gamma) = \frac{1}{2}(\sin 2\beta - \sin 2\gamma)$$

$$\cos(\gamma+\alpha)\sin(\gamma-\alpha) = \frac{1}{2}(\sin 2\gamma - \sin 2\alpha)$$

よって, 与式$=0$

(2)

$$\cos\alpha\sin(\beta-\gamma)=\frac{1}{2}[\sin\{\alpha+(\beta-\gamma)\}-\sin\{\alpha-(\beta-\gamma)\}]$$
$$=\frac{1}{2}\{\sin(\alpha+\beta-\gamma)-\sin(\alpha-\beta+\gamma)\}$$

同様にして，

$$\cos\beta\sin(\gamma-\alpha)=\frac{1}{2}[\sin\{\beta+(\gamma-\alpha)\}-\sin\{\beta-(\gamma-\alpha)\}]$$
$$=\frac{1}{2}\{\sin(-\alpha+\beta+\gamma)-\sin(\alpha+\beta-\gamma)\}$$

$$\cos\gamma\sin(\alpha-\beta)=\frac{1}{2}[\sin\{\gamma+(\alpha-\beta)\}-\sin\{\gamma-(\alpha-\beta)\}]$$
$$=\frac{1}{2}\{\sin(\alpha-\beta+\gamma)-\sin(-\alpha+\beta+\gamma)\}$$

よって，与式＝0

309

解法 1：和積の公式を利用

$$\cos x+\cos 3x=2\cos\frac{x+3x}{2}\cos\frac{x-3x}{2}$$
$$=2\cos 2x\cos(-x)$$
$$=2\cos 2x\cos x$$

より，

$2\cos 2x\cos x=0$

よって，$\cos 2x=0$ または $\cos x=0$

これと $0\leq x<2\pi$ より，$x=\frac{\pi}{4},\ \frac{\pi}{2},\ \frac{3}{4}\pi,\ \frac{5}{4}\pi,\ \frac{3}{2}\pi,\ \frac{7}{4}\pi$

解法 2：**3** 倍角の公式を利用

$$\cos x+\cos 3x=\cos x+(-3\cos x+4\cos^3 x)$$
$$=4\cos^3 x-2\cos x$$
$$=2\cos x(2\cos^2 x-1)$$

より，

$2\cos x(2\cos^2 x-1)=0$

よって，$\cos x=0$ または $\cos^2 x=\frac{1}{2}$

これと $0\leq x<2\pi$ より，$x=\frac{\pi}{4},\ \frac{\pi}{2},\ \frac{3}{4}\pi,\ \frac{5}{4}\pi,\ \frac{3}{2}\pi,\ \frac{7}{4}\pi$

310

$$\begin{aligned}\cos x+\cos 3x+\cos 5x&=\cos x+2\cos\frac{3x+5x}{2}\cos\frac{3x-5x}{2}\\&=\cos x+2\cos 4x\cos(-x)\\&=\cos x+2\cos 4x\cos x\\&=\cos x(1+2\cos 4x)\end{aligned}$$

より,

$\cos x(1+2\cos 4x)<0$

$0\leq x<\dfrac{\pi}{2}$ だから, $\cos x>0$

よって, $1+2\cos 4x<0$, すなわち $\cos 4x<-\dfrac{1}{2}$, $0\leq 4x<2\pi$

ゆえに, $\dfrac{2}{3}\pi<4x<\dfrac{4}{3}\pi$, すなわち $\dfrac{\pi}{6}<x<\dfrac{\pi}{3}$

311

$$\begin{aligned}\sin x\sin\left(x+\frac{\pi}{3}\right)&=\frac{1}{2}\left[\cos\left\{x-\left(x+\frac{\pi}{3}\right)\right\}-\cos\left\{x+\left(x+\frac{\pi}{3}\right)\right\}\right]\\&=\frac{1}{2}\left\{\cos\frac{\pi}{3}-\cos\left(2x+\frac{\pi}{3}\right)\right\}\\&=\frac{1}{2}\left\{\frac{1}{2}-\cos\left(2x+\frac{\pi}{3}\right)\right\}\end{aligned}$$

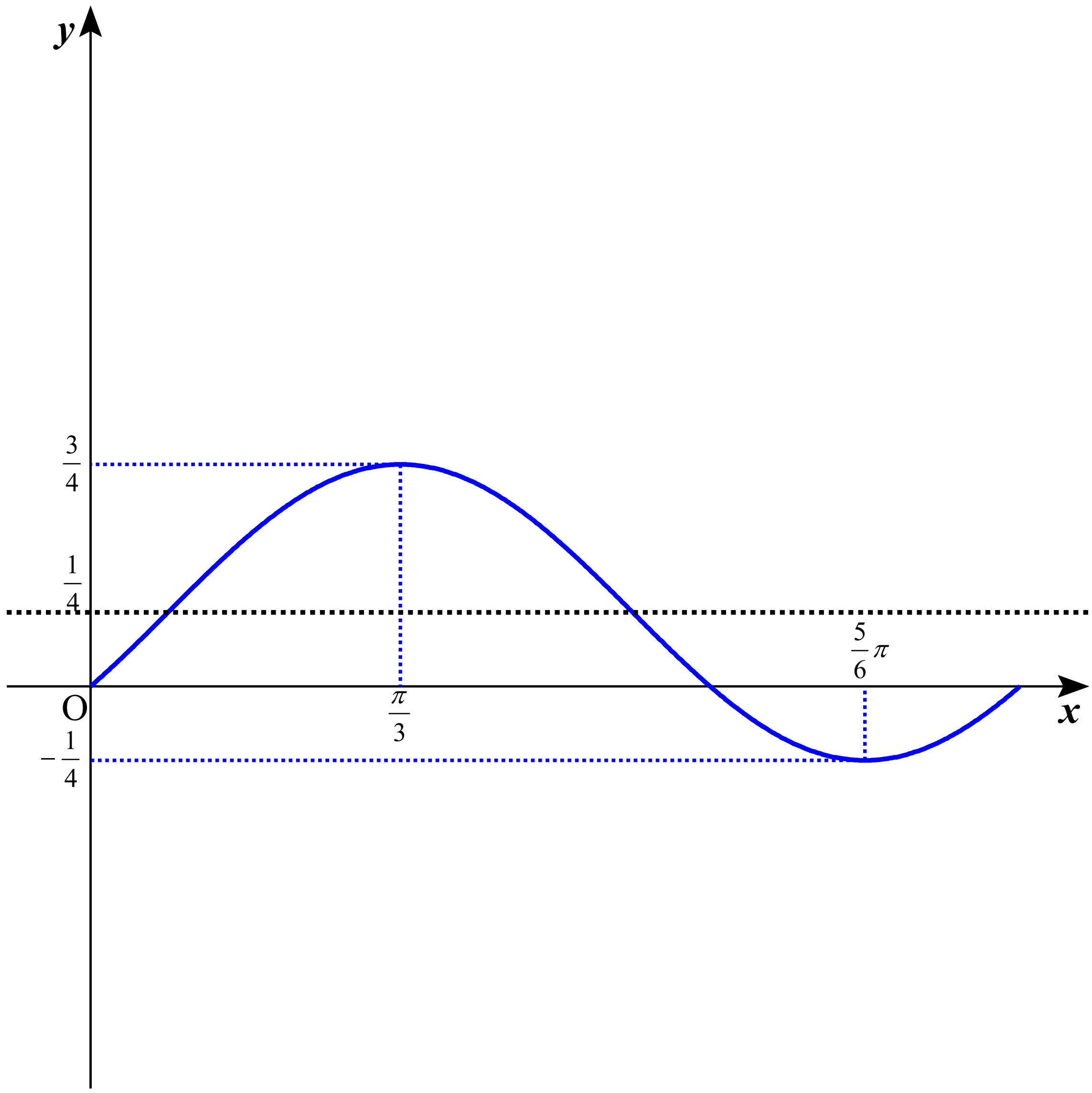
より,

$y=\dfrac{1}{2}\left\{\dfrac{1}{2}-\cos\left(2x+\dfrac{\pi}{3}\right)\right\}$

これと $\dfrac{\pi}{3}\leq 2x+\dfrac{\pi}{3}\leq 2\pi+\dfrac{\pi}{3}$ より,

$2x+\dfrac{\pi}{3}=\pi$, すなわち $x=\dfrac{\pi}{3}$ のとき最大値 $\dfrac{3}{4}$

$2x+\dfrac{\pi}{3}=2\pi$, すなわち $x=\dfrac{5}{6}\pi$ のとき最小値 $-\dfrac{1}{4}$


y
$\frac{3}{4}$
$\frac{1}{4}$
O
$-\frac{1}{4}$
$\frac{\pi}{3}$
$\frac{5}{6}\pi$
x

312

$$
\begin{aligned}
\text{左辺} &= \sin 2A + \sin 2B + \sin 2C \\
&= 2\sin\frac{2A+2B}{2}\cos\frac{2A-2B}{2} + \sin 2\{\pi - (A+B)\} \\
&= 2\sin(A+B)\cos(A-B) + \sin\{2\pi - 2(A+B)\} \\
&= 2\sin(A+B)\cos(A-B) - \sin\{2(A+B)\} \\
&= 2\sin(A+B)\cos(A-B) - 2\sin(A+B)\cos(A+B) \\
&= 2\sin(A+B)\{\cos(A-B) - \cos(A+B)\} \\
&= 2\sin\{\pi - (A+B)\}\cdot 2\sin A\sin B \\
&= 4\sin C\sin A\sin B \\
&= \text{右辺}
\end{aligned}
$$

$$
\begin{aligned}
\text{左辺} &= \sin 2A + \sin 2B + \sin 2C \\
&= \sin 2A + \sin 2B + \sin 2\{\pi - (A+B)\} \\
&= \sin 2A + \sin 2B + \sin\{2\pi - 2(A+B)\} \\
&= \sin 2A + \sin 2B - \sin(2A+2B) \\
&= \sin 2A + \sin 2B - (\sin 2A\cos 2B + \sin 2B\cos 2A) \\
&= \sin 2A(1-\cos 2B) + \sin 2B(1-\cos 2A) \\
&= \sin 2A\cdot 2\sin^2 B + \sin 2B\cdot 2\sin^2 A \\
&= 4\sin A\cos A\sin^2 B + 4\sin B\cos B\sin^2 A \\
&= 4\sin A\sin B(\sin B\cos A + \sin A\cos B) \\
&= 4\sin A\sin B\sin(A+B) \\
&= 4\sin A\sin B\sin\{\pi - (A+B)\} \\
&= 4\sin A\sin B\sin C \\
&= \text{右辺}
\end{aligned}
$$

など