指数関数と対数関数 1　指数の拡張

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置き換えると計算が楽

(1)

$a^{\frac{1}{4}}=s,\ b^{\frac{1}{4}}=t$ とおくと,

$$\begin{aligned}\left(s^2+st+t^2\right)\left(s^2-st+t^2\right)&=\left\{\left(s^2+t^2\right)+st\right\}\left\{\left(s^2+t^2\right)-st\right\}\\&=\left(s^2+t^2\right)^2-s^2t^2\\&=s^4+s^2t^2+t^4\end{aligned}$$

$$\therefore a+a^{\frac{1}{2}}b^{\frac{1}{2}}+b$$

(2)

$a^{\frac{x}{3}}=s,\ b^{-\frac{x}{3}}=t$ とおくと, $(s-t)\left(s^2+st+t^2\right)=s^3-t^3 \quad \therefore a^x-b^{-x}$

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累乗根は指数にして計算すると楽

(1)

$$\begin{aligned}\left(6^{\frac{1}{4}}+5^{\frac{1}{4}}\right)\left(6^{\frac{1}{4}}-5^{\frac{1}{4}}\right)&=\left(6^{\frac{1}{4}}\right)^2-\left(5^{\frac{1}{4}}\right)^2\\&=6^{\frac{1}{2}}-5^{\frac{1}{2}}\\&=\sqrt{6}-\sqrt{5}\end{aligned}$$

(2)

$$\begin{aligned}\text{与式}&=\left(5^{\frac{1}{3}}+3^{\frac{1}{3}}\right)\left\{\left(5^{\frac{1}{3}}\right)^2-5^{\frac{1}{3}}3^{\frac{1}{3}}+\left(3^{\frac{1}{3}}\right)^2\right\}\\&=\left(5^{\frac{1}{3}}\right)^3+\left(3^{\frac{1}{3}}\right)^3\\&=5+3\\&=8\end{aligned}$$

(3)

$$与式=\left(4^{\frac{1}{3}}+2^{\frac{1}{3}}\right)^3+\left(4^{\frac{1}{3}}-2^{\frac{1}{3}}\right)^3$$
$$=\left\{\left(4^{\frac{1}{3}}+2^{\frac{1}{3}}\right)+\left(4^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\right\}\left\{\left(4^{\frac{1}{3}}+2^{\frac{1}{3}}\right)^2-\left(4^{\frac{1}{3}}+2^{\frac{1}{3}}\right)\left(4^{\frac{1}{3}}-2^{\frac{1}{3}}\right)+\left(4^{\frac{1}{3}}-2^{\frac{1}{3}}\right)^2\right\}$$
$$=2\cdot4^{\frac{1}{3}}\left(4^{\frac{2}{3}}+3\cdot2^{\frac{2}{3}}\right)$$
$$=2\cdot4^{\frac{1}{3}}\cdot4^{\frac{2}{3}}+6\cdot4^{\frac{1}{3}}\cdot2^{\frac{2}{3}}$$
$$=2\cdot4+6\cdot2^{\frac{2}{3}}\cdot2^{\frac{2}{3}}$$
$$=8+6\cdot2^{1+\frac{1}{3}}$$
$$=8+6\cdot2\cdot2^{\frac{1}{3}}$$
$$=8+12\sqrt[3]{2}$$

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負の数の累乗根では，-1 を切り離して処理することから始めると楽かも？

(1)

$$\sqrt[3]{-216}=\sqrt[3]{-1\cdot2^3\cdot3^3}$$
$$=\sqrt[3]{(-1)^3\cdot2^3\cdot3^3}$$
$$=\sqrt[3]{(-1\cdot2\cdot3)^3}$$
$$=\sqrt[3]{(-6)^3}$$
$$=-6$$

(2)

$$\sqrt[5]{-32}=\sqrt[5]{-1\cdot2^5}$$
$$=\sqrt[5]{(-1)^5\cdot2^5}$$
$$=\sqrt[5]{(-2)^5}$$
$$=-2$$

(3)

$$\sqrt[3]{-\frac{1}{64}}=\sqrt[3]{-1\cdot\left(\frac{1}{4}\right)^3}$$
$$=\sqrt[3]{(-1)^3\left(\frac{1}{4}\right)^3}$$
$$=\sqrt[3]{\left(-\frac{1}{4}\right)^3}$$
$$=-\frac{1}{4}$$

(4)

$$\sqrt[3]{3^3\cdot 2}\times\sqrt[3]{-1\cdot 2}\times\sqrt[3]{2^4}=\sqrt[3]{3^3}\cdot\sqrt[3]{2}\times\sqrt[3]{(-1)^3\cdot 2}\times\sqrt[3]{2^3\cdot 2}$$
$$=3\sqrt[3]{2}\times\sqrt[3]{(-1)^3}\cdot\sqrt[3]{2}\times\sqrt[3]{2^3}\cdot\sqrt[3]{2}$$
$$=3\sqrt[3]{2}\times(-1)\cdot\sqrt[3]{2}\times 2\sqrt[3]{2}$$
$$=-6\cdot\left(\sqrt[3]{2}\right)^3$$
$$=-6\cdot\sqrt[3]{2^3}$$
$$=-6\cdot 2$$
$$=-12$$

(5)

$$\sqrt[3]{(-1)\cdot 2^3\cdot 3}+\sqrt[3]{3^4}+\sqrt[3]{-1\cdot 3}=\sqrt[3]{(-1)^3\cdot 2^3\cdot 3}+\sqrt[3]{3^3\cdot 3}+\sqrt[3]{(-1)^3\cdot 3}$$
$$=\sqrt[3]{(-2)^3\cdot 3}+\sqrt[3]{3^3}\cdot\sqrt[3]{3}+\sqrt[3]{(-1)^3}\cdot\sqrt[3]{3}$$
$$=\sqrt[3]{(-2)^3}\cdot\sqrt[3]{3}+3\sqrt[3]{3}+(-1)\cdot\sqrt[3]{3}$$
$$=-2\sqrt[3]{3}+3\sqrt[3]{3}-\sqrt[3]{3}$$
$$=(-2+3-1)\sqrt[3]{3}$$
$$=0$$

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$$x+x^{-1}=\left(x^{\frac{1}{3}}\right)^3+\left(x^{-\frac{1}{3}}\right)^3$$
$$=\left(x^{\frac{1}{3}}+x^{-\frac{1}{3}}\right)^3-3x^{\frac{1}{3}}x^{-\frac{1}{3}}\left(x^{\frac{1}{3}}+x^{-\frac{1}{3}}\right)$$
$$=3^3-3\cdot 1\cdot 3$$
$$=18$$

$$x^3+x^{-3}=x^3+\left(x^{-1}\right)^3$$
$$=\left(x+x^{-1}\right)^3-3x\cdot x^{-1}\left(x+x^{-1}\right)$$
$$=18^3-3\cdot 1\cdot 18$$
$$=5778$$

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$$\left(a^{4x}-a^{-4x}\right)\div\left(a^x-a^{-x}\right)=\left\{\left(a^{2x}\right)^2-\left(a^{-2x}\right)^2\right\}\div\left(a^x-a^{-x}\right)$$
$$=\left(a^{2x}-a^{-2x}\right)\left(a^{2x}+a^{-2x}\right)\div\left(a^x-a^{-x}\right)$$
$$=\left\{\left(a^x\right)^2-\left(a^{-x}\right)^2\right\}\left(a^{2x}+\frac{1}{a^{2x}}\right)\div\left(a^x-a^{-x}\right)$$
$$=\left(a^x-a^{-x}\right)\left(a^x+a^{-x}\right)\left(5+\frac{1}{5}\right)\div\left(a^x-a^{-x}\right)$$
$$=\left(a^x+\frac{1}{a^x}\right)\cdot\frac{26}{5}$$

ここで，$a>0$ より，$a^x>0$ だから，$a^x=\left(a^{2x}\right)^{\frac{1}{2}}=\sqrt{5}$

よって，

$$\left(a^{4x}-a^{-4x}\right)\div\left(a^x-a^{-x}\right)=\left(a^x+\frac{1}{a^x}\right)\cdot\frac{26}{5}$$
$$=\left(\sqrt{5}+\frac{1}{\sqrt{5}}\right)\cdot\frac{26}{5}$$
$$=\frac{6\sqrt{5}}{5}\cdot\frac{26}{5}$$
$$=\frac{156\sqrt{5}}{25}$$

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$$\begin{aligned}\left(2^{x}+2^{-x}\right)^{2}&=\left(2^{x}\right)^{2}+2\cdot 2^{x}\cdot 2^{-x}+\left(2^{-x}\right)^{2}\\&=\left(2^{x}\right)^{2}-2\cdot 2^{x}\cdot 2^{-x}+\left(2^{-x}\right)^{2}+4\cdot 2^{x}\cdot 2^{-x}\\&=\left(2^{x}-2^{-x}\right)^{2}+4\cdot 2^{x}\cdot 2^{-x}\\&=3^{2}+4\cdot 1\\&=13\end{aligned}$$

これと $2^{x}>0,\ 2^{-x}>0$ より，$2^{x}+2^{-x}=\sqrt{13}$