積分法 2　置換積分法

コツ

できるだけ大きい式のかたまりを置換すると式処理が楽。

377

(1)

$\sqrt[3]{1+x}=t$ とおくと，$1+x=t^3$ より，$\dfrac{dx}{dt}=3t^2 \quad \therefore dx=3t^2dt$

また，$x=t^3-1$

$$\begin{aligned}\therefore \int x\sqrt[3]{1+x}dx &= \int \left(t^3-1\right)t\cdot 3t^2dt\\ &=3\int\left(t^6-t^3\right)dt\\ &=3\left(\frac{1}{7}t^7-\frac{1}{4}t^4\right)+C\\ &=\frac{3}{28}t^4\left(4t^3-7\right)+C\\ &=\frac{3}{28}(1+x)\sqrt[3]{1+x}\{4(1+x)-7\}+C\\ &=\frac{3}{28}(4x-3)(1+x)\sqrt[3]{1+x}+C\end{aligned}$$

(2)

$\cos x=t$ とおくと，$-\sin x=\dfrac{dt}{dx}$ より，$\sin xdx=-dt$

$$\begin{aligned}\therefore \int \sin x\cos^4 xdx &= \int \cos^4 x\sin xdx\\ &=-\int t^4dt\\ &=-\frac{1}{5}t^5+C\\ &=-\frac{1}{5}\cos^5 x+C\end{aligned}$$

(3)

$\tan x=t$ とおくと，$1+t^2=\dfrac{1}{\cos^2 x}$，$\dfrac{1}{\cos^2 x}=\dfrac{dt}{dx}$ より，$\dfrac{dx}{\cos^2 x}=dt$

$$\begin{aligned}\therefore \int \frac{dx}{\cos^4 x} &= \int \frac{1}{\cos^2 x}\frac{dx}{\cos^2 x}\\ &=\int\left(1+t^2\right)dt\\ &=t+\frac{1}{3}t^3+C\\ &=\tan x+\frac{1}{3}\tan^3 x+C\end{aligned}$$

(4)

$e^{x^2+x+5}=t$ とおくと，$(2x+1)e^{x^2+x+5}=\dfrac{dt}{dx}$ より，$(2x+1)e^{x^2+x+5}dx=dt$

$$\begin{aligned}\therefore \int (2x+1)e^{x^2+x+5}dx &= \int dt \\ &= t+C \\ &= e^{x^2+x+5}+C\end{aligned}$$

(5)

$e^x+2=t$ とおくと，$e^x dx=dt$

$$\begin{aligned}\therefore \int \frac{e^{2x}}{(e^x+2)^2}dx &= \int \frac{e^x}{(e^x+2)^2}e^x dx \\ &= \int \frac{t-2}{t^2}dt \\ &= \int \left(\frac{1}{t}-2t^{-2}\right)dt \\ &= \log|t|+2t^{-1}+C \\ &= \log(e^x+2)+\frac{2}{e^x+2}+C\end{aligned}$$

(6)

$\log x-1=t$ とおくと，$\dfrac{dx}{x}=dt$

$$\begin{aligned}\therefore \int \frac{\log x}{x(\log x-1)^2}dx &= \int \frac{\log x}{(\log x-1)^2}\frac{dx}{x} \\ &= \int \frac{t+1}{t^2}dt \\ &= \int \left(\frac{1}{t}+t^{-2}\right)dt \\ &= \log|t|-\frac{1}{t}+C \\ &= \log|\log x-1|-\frac{1}{\log x-1}+C\end{aligned}$$