## 積分法 3　部分積分法

ポイント

部分積分といえば，次数下げと周期性

**380**

**(1)**

解法 1：部分積分だけで解く

$$\int x\log(x^2-2)dx=\int\left(\frac{x^2-2}{2}\right)'\log(x^2-2)dx$$
$$=\frac{1}{2}(x^2-2)\log(x^2-2)-\int\frac{x^2-2}{2}\{\log(x^2-2)\}'dx$$
$$=\frac{1}{2}(x^2-2)\log(x^2-2)-\int\frac{x^2-2}{2}\cdot\frac{2x}{x^2-2}dx$$
$$=\frac{1}{2}(x^2-2)\log(x^2-2)-\int xdx$$
$$=\frac{1}{2}\{(x^2-2)\log(x^2-2)-x^2\}+C$$

解法 2：置換積分＋部分積分で解く

$x^2-2=t$ とおくと， $xdx=\dfrac{dt}{2}$

$$\therefore\int x\log(x^2-2)dx=\int\log(x^2-2)xdx$$
$$=\int\log t\frac{dt}{2}$$
$$=\frac{1}{2}\int\log tdt$$
$$=\frac{1}{2}\int t'\log tdt$$
$$=\frac{1}{2}\left\{t\log t-\int t(\log t)'dt\right\}$$
$$=\frac{1}{2}\left(t\log t-\int dt\right)$$
$$=\frac{1}{2}(t\log t-t)+C'$$
$$=\frac{1}{2}\{(x^2-2)\log(x^2-2)-(x^2-2)\}+C'$$
$$=\frac{1}{2}\{(x^2-2)\log(x^2-2)-x^2\}+1+C'$$
$$=\frac{1}{2}\{(x^2-2)\log(x^2-2)-x^2\}+C$$

**(2)**

解法 **1**：部分積分だけで解く

$$\int e^x \log(e^x+1)dx = \int (e^x+1)' \log(e^x+1)dx$$
$$= (e^x+1)\log(e^x+1) - \int (e^x+1)\{\log(e^x+1)\}' dx$$
$$= (e^x+1)\log(e^x+1) - \int (e^x+1)\frac{e^x}{e^x+1} dx$$
$$= (e^x+1)\log(e^x+1) - \int e^x dx$$
$$= (e^x+1)\log(e^x+1) - e^x + C$$

解法 **2**：置換積分＋部分積分で解く

$e^x+1=t$ とおくと， $e^x dx = dt$

$$\therefore \int e^x \log(e^x+1)dx = \int \log t dt$$
$$= \int t' \log t dt$$
$$= t\log t - \int t(\log t)' dt$$
$$= t\log t - \int t\cdot\frac{1}{t} dt$$
$$= t\log t - \int dt$$
$$= t\log t - t + C'$$
$$= (e^x+1)\log(e^x+1) - (e^x+1) + C'$$
$$= (e^x+1)\log(e^x+1) - e^x + C$$

**381**

**(1)**

$$\int x^2 \sin 2x dx = \int x^2\left(-\frac{\cos 2x}{2}\right)' dx$$
$$= -\frac{x^2}{2}\cos 2x - \int (x^2)'\left(-\frac{\cos 2x}{2}\right)dx$$
$$= -\frac{x^2}{2}\cos 2x + \int x\cos 2x dx$$
$$= -\frac{x^2}{2}\cos 2x + \int x\left(\frac{\sin 2x}{2}\right)' dx$$
$$= -\frac{x^2}{2}\cos 2x + \frac{x}{2}\sin 2x - \int x'\left(\frac{\sin 2x}{2}\right)dx$$
$$= -\frac{x^2}{2}\cos 2x + \frac{x}{2}\sin 2x - \int \frac{\sin 2x}{2} dx$$
$$= -\frac{x^2}{2}\cos 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + C$$

**(2)**

$$\int x^2 e^{-x} dx = \int x^2 \left(-e^{-x}\right)' dx$$
$$= x^2\left(-e^{-x}\right) - \int \left(x^2\right)'\left(-e^{-x}\right)dx$$
$$= -x^2 e^{-x} + 2\int xe^{-x}dx$$
$$= -x^2 e^{-x} + 2\int x\left(-e^{-x}\right)' dx$$
$$= -x^2 e^{-x} + 2\left\{x\left(-e^{-x}\right) - \int x'\left(-e^{-x}\right)dx\right\}$$
$$= -x^2 e^{-x} + 2\left(-xe^{-x} + \int e^{-x}dx\right)$$
$$= -x^2 e^{-x} + 2\left(-xe^{-x} - e^{-x}\right) + C$$
$$= -\left(x^2 + 2x + 2\right)e^{-x} + C$$

**(3)**

解法 **1**：部分積分だけで解く

$$\int (\log x)^3 dx = \int x'(\log x)^3 dx$$
$$= x(\log x)^3 - \int x\left\{(\log x)^3\right\}' dx$$
$$= x(\log x)^3 - \int x\left\{3(\log x)'(\log x)^2\right\}dx$$
$$= x(\log x)^3 - 3\int x \cdot \frac{1}{x}(\log x)^2 dx$$
$$= x(\log x)^3 - 3\int (\log x)^2 dx$$
$$= x(\log x)^3 - 3\int x'(\log x)^2 dx$$
$$= x(\log x)^3 - 3\left[x(\log x)^2 - \int x\left\{(\log x)^2\right\}' dx\right]$$
$$= x(\log x)^3 - 3x(\log x)^2 + 3\int x\left\{2(\log x)' \log x\right\}dx$$
$$= x(\log x)^3 - 3x(\log x)^2 + 6\int x \cdot \frac{1}{x}\log x dx$$
$$= x(\log x)^3 - 3x(\log x)^2 + 6\int \log x dx$$
$$= x(\log x)^3 - 3x(\log x)^2 + 6\int x' \log x dx$$
$$= x(\log x)^3 - 3x(\log x)^2 + 6\left\{x\log x - \int x(\log x)' dx\right\}$$
$$= x(\log x)^3 - 3x(\log x)^2 + 6x\log x - 6\int x \cdot \frac{1}{x}dx$$
$$= x(\log x)^3 - 3x(\log x)^2 + 6x\log x - 6\int dx$$
$$= x\left\{(\log x)^3 - 3(\log x)^2 + 6\log x - 6\right\} + C$$

解法 **2**：置換積分＋部分積分で解く

$\log x = t$ とおくと， $x = e^t$ ， $dx = e^t dt$

$$\therefore \int (\log x)^3 dx = \int t^3 e^t dt$$
$$= \int t^3 (e^t)' dt$$
$$= t^3 e^t - \int (t^3)' e^t dt$$
$$= t^3 e^t - 3\int t^2 e^t dt$$
$$= t^3 e^t - 3\int t^2 (e^t)' dt$$
$$= t^3 e^t - 3\left\{t^2 e^t - \int (t^2)' e^t dt\right\}$$
$$= t^3 e^t - 3t^2 e^t + 6\int te^t dt$$
$$= t^3 e^t - 3t^2 e^t + 6\int t(e^t)' dtx$$
$$= t^3 e^t - 3t^2 e^t + 6\left(te^t - \int t' e^t dt\right)$$
$$= t^3 e^t - 3t^2 e^t + 6te^t - 6\int e^t dt$$
$$= e^t (t^3 - 3t^2 + 6t - 6) + C$$
$$= x\left\{(\log x)^3 - 3(\log x)^2 + 6\log x - 6\right\} + C$$