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$y=f(x)=\dfrac{1}{x+\sqrt{2-x}}$ とおくと，

導関数の定義（平均変化率の極限）より，

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{f(x+h)-f(x)}{h}=\frac{\dfrac{1}{x+h+\sqrt{2-x-h}}-\dfrac{1}{x+\sqrt{2-x}}}{h}$$

$$=\frac{\dfrac{1}{x+h+\sqrt{2-x-h}}-\dfrac{1}{x+\sqrt{2-x}}}{h}\times\frac{\left(x+h+\sqrt{2-x-h}\right)\left(x+\sqrt{2-x}\right)}{\left(x+h+\sqrt{2-x-h}\right)\left(x+\sqrt{2-x}\right)}$$

$$=\frac{\sqrt{2-x}-\sqrt{2-x-h}-h}{h\left(x+h+\sqrt{2-x-h}\right)\left(x+\sqrt{2-x}\right)}$$

$$=\frac{1}{\left(x+h+\sqrt{2-x-h}\right)\left(x+\sqrt{2-x}\right)}\left(\frac{\sqrt{2-x}-\sqrt{2-x-h}}{h}-1\right)$$

$$=\frac{1}{\left(x+h+\sqrt{2-x-h}\right)\left(x+\sqrt{2-x}\right)}\left(\frac{1}{\sqrt{2-x}+\sqrt{2-x-h}}-1\right)$$

よって，

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\frac{1-2\sqrt{2-x}}{2\sqrt{2-x}\left(x+\sqrt{2-x}\right)^2}$$