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$$y = f^{-1}(x) \text{ より, } x = f(y)$$

$$\text{両辺を } x \text{ で微分すると, } 1 = \frac{df(y)}{dx}$$

$$\text{ここで, } \frac{df(y)}{dx} = \frac{df(y)}{dy} \cdot \frac{dy}{dx} = f'(y) \frac{dy}{dx}$$

$$\text{よって, } \frac{dy}{dx} = \frac{1}{f'(y)}$$

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{d\left(\frac{dy}{dx}\right)}{dx} \\ &= \frac{d\left(\frac{1}{f'(y)}\right)}{dx} \\ &= \frac{d\left(\frac{1}{f'(y)}\right)}{dy} \cdot \frac{dy}{dx} \\ &= \frac{-f''(y)}{\{f'(y)\}^2} \cdot \frac{1}{f'(y)} \\ &= -\frac{f''(y)}{\{f'(y)\}^3} \end{aligned}$$