

74

$$y = g(x) \text{ より, } g'(x) = \frac{dy}{dx} \quad \dots \textcircled{1}$$

$$g(x) = f^{-1}(x) \text{ のとおくと, } y = f^{-1}(x) \quad \therefore x = f(y)$$

$$\therefore \frac{dx}{dy} = \frac{df(y)}{dy} = f'(y) \quad \therefore \frac{dy}{dx} = \frac{1}{f'(y)} \quad \dots \textcircled{2}$$

$$\textcircled{1}, \textcircled{2} \text{ より, } g'(x) = \frac{1}{f'(y)}$$

$$\text{また, } g''(x) = \frac{d^2y}{dx^2} \quad \dots \textcircled{3}$$

一方, $\textcircled{2}$ より,

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{d}{dx} \left(\frac{dy}{dx} \right) \\ &= \frac{d}{dx} \left(\frac{1}{f'(y)} \right) \\ &= \frac{d}{dy} \left(\frac{1}{f'(y)} \right) \cdot \frac{dy}{dx} \\ &= \frac{-f''(y)}{\{f'(y)\}^2} \cdot \frac{1}{f'(y)} \\ &= -\frac{f''(y)}{\{f'(y)\}^3} \quad \dots \textcircled{4} \end{aligned}$$

 $\textcircled{3}, \textcircled{4}$ より,

$$g''(x) = -\frac{f''(y)}{\{f'(y)\}^3}$$

ここで,

$$y = g(x) = f^{-1}(x), \quad x = f(y), \quad 2 = f(1) \text{ より, } x = 2, y = 1$$

よって,

$$g''(2) = -\frac{f''(1)}{\{f'(1)\}^3} = -\frac{3}{8}$$