115

テキスト解説補完

置換積分で解く場合

$\log x = t$ とおき，両辺を x について微分すると，$\dfrac{d\log x}{dx} = \dfrac{dt}{dx}$　$\therefore \dfrac{1}{x} = \dfrac{dt}{dx}$

$\therefore dx = xdt$

これと $x = e^t$ より，$dx = e^t dt$

$$\therefore \int 5^{\log x} dx = \int 5^t e^t dt = \int (5e)^t dt$$

ここで，$a = e^{\log a}$ より，$a = (5e)^t$ とおくと，$(5e)^t = e^{\log(5e)^t} = e^{t\log 5e}$

よって，

$$\int (5e)^t dt = \int e^{t\log 5e} dt = \frac{1}{\log 5e} e^{t\log 5e} + C$$

$$= \frac{1}{1+\log 5}(5e)^t + C \quad \left(\because e^{t\log 5e} = (5e)^t\right)$$

$$\therefore \int_1^e 5^{\log x} dx = \int_0^1 (5e)^t dt = \frac{1}{1+\log 5}\left[(5e)^t\right]_0^1 = \frac{5e-1}{1+\log 5}$$

直接解く場合

$a = e^{\log a}$ より，$a = 5^{\log x}$ とおくと，

$$5^{\log x} = e^{\log 5^{\log x}} = e^{\log x \log 5} = \left(e^{\log x}\right)^{\log 5} = x^{\log 5}$$

⇧ $\left(\because x = e^{\log x}\right)$

よって，

$$\int_1^e 5^{\log x} dx = \int_1^e x^{\log 5} dx = \frac{1}{1+\log 5}\left[x^{1+\log 5}\right]_1^e = \frac{1}{1+\log 5}\left(e^{1+\log 5} - 1\right) = \frac{e^{\log 5}\cdot e - 1}{1+\log 5} = \frac{5e-1}{1+\log 5}$$

⇧ $\left(\because e^{1+\log 5} = e \cdot e^{\log 5}\right)$

⇧ $\left(\because 5 = e^{\log 5}\right)$