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正面突破してみる。

$$\begin{aligned}
\frac{\sin\theta}{\sin\theta+\cos\theta} &= \frac{\sin\theta(\cos\theta-\sin\theta)}{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)} \\
&= \frac{\sin\theta\cos\theta-\sin^2\theta}{\cos^2\theta-\sin^2\theta} \\
&= \frac{\dfrac{\sin 2\theta}{2}-\dfrac{1-\cos 2\theta}{2}}{\cos 2\theta} \\
&= \frac{1}{2}\cdot\frac{\sin 2\theta+\cos 2\theta-1}{\cos 2\theta} \\
&= \frac{1}{2}\left(\frac{\sin 2\theta}{\cos 2\theta}+1-\frac{1}{\cos 2\theta}\right) \\
&= \frac{1}{2}\left\{-\frac{1}{2}\cdot\frac{(\cos 2\theta)'}{\cos 2\theta}+1-\frac{\cos 2\theta}{\cos^2\theta}\right\} \\
&= \frac{1}{2}\left\{-\frac{1}{2}\cdot\frac{(\cos 2\theta)'}{\cos 2\theta}+1-\frac{\cos 2\theta}{1-\sin^2 2\theta}\right\} \\
&= \frac{1}{2}\left\{-\frac{1}{2}\cdot\frac{(\cos 2\theta)'}{\cos 2\theta}+1-\frac{\cos 2\theta}{1-\sin^2 2\theta}\right\} \\
&= \frac{1}{2}\left\{-\frac{1}{2}\cdot\frac{(\cos 2\theta)'}{\cos 2\theta}+1-\frac{1}{2}\left(\frac{\cos 2\theta}{1-\sin 2\theta}+\frac{\cos 2\theta}{1+\sin 2\theta}\right)\right\} \\
&= \frac{1}{2}\left[-\frac{1}{2}\cdot\frac{(\cos 2\theta)'}{\cos 2\theta}+1-\frac{1}{2}\left\{-\frac{1}{2}\cdot\frac{(1-\sin 2\theta)'}{1-\sin 2\theta}+\frac{1}{2}\cdot\frac{(1+\cos 2\theta)'}{1+\sin 2\theta}\right\}\right] \\
&= -\frac{1}{4}\cdot\frac{(\cos 2\theta)'}{\cos 2\theta}+\frac{1}{2}+\frac{1}{8}\left\{\frac{(1-\sin 2\theta)'}{1-\sin 2\theta}-\frac{(1+\cos 2\theta)'}{1+\sin 2\theta}\right\}
\end{aligned}$$

よって，

$$\begin{aligned}
\int_0^{\frac{\pi}{2}}\frac{\sin\theta}{\sin\theta+\cos\theta}d\theta &= \left[-\frac{1}{4}\log|\cos 2\theta|+\frac{\theta}{2}+\frac{1}{8}\log\left|\frac{1-\sin 2\theta}{1+\sin 2\theta}\right|\right]_0^{\frac{\pi}{2}} \\
&= \frac{\pi}{4}
\end{aligned}$$