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(2)

別解

 $\log x = t$ とおくと, $x = e^t$ より, $dx = e^t dt$

$$\begin{aligned}\int (\log x)^2 dx &= \int t^2 e^t dt \\ &= (t^2 - 2t + 2) \cdot e^t + C \\ &= \{(\log x)^2 - 2 \log x + 2\} \cdot x + C \\ &= x(\log x)^2 - 2x \log x + 2x + C\end{aligned}$$

(3)

別解

 $\log x = t$ とおくと, $x = e^t$ より, $dx = e^t dt$

$$\begin{aligned}\int \frac{\log x}{x} dx &= \int \frac{t}{e^t} \cdot e^t dt \\ &= \int t dt \\ &= \frac{1}{2} t^2 + C \\ &= \frac{1}{2} (\log x)^2 + C\end{aligned}$$

(4)

別解

 $\log x = t$ とおくと, $x = e^t$ より, $dx = e^t dt$

$$\begin{aligned}\int x \log x dx &= \int t e^{2t} dt \\ &= \frac{t e^{2t}}{2} - \int \frac{e^{2t}}{2} dt \\ &= \frac{t e^{2t}}{2} - \frac{e^{2t}}{4} + C \\ &= \frac{x^2 \log x}{2} - \frac{x^2}{4} + C\end{aligned}$$

(5)

別解

 $\log x = t$ とおくと, $x = e^t$ より, $dx = e^t dt$

$$\begin{aligned}\int x(\log x)^2 dx &= \int t^2 e^t \cdot e^t dt \\&= \int t^2 e^{2t} dt \\&= \left(\frac{t^2}{2} - \frac{2t}{4} + \frac{2}{8} \right) \cdot e^{2t} + C \\&= \left\{ \frac{(\log x)^2}{2} - \frac{\log x}{2} + \frac{1}{4} \right\} x^2 + C \\&= \frac{x^2 (\log x)^2}{2} - \frac{x^2 \log x}{2} + \frac{x^2}{4} + C\end{aligned}$$